Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #12

Integration by Substitution – Examples

Example 1:

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Let $u = \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}dx$ so $2du = \frac{1}{\sqrt{x}}dx$.
Hence,

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = 2\int \sin u\,du$$

$$= -2\cos u + C = -2\cos\sqrt{x} + C$$

Example 2:

$$\int (\cos^3 x)\sin x\,dx$$

Let $u = \cos x$ so that $du = -\sin x\,dx$. Then

$$\int (\cos^3 x)\sin x\,dx = -\int u^3\,du$$

$$= -\frac{1}{4}u^4 + C = -\frac{1}{4}\cos^4 x + C$$

Example 3:

$$\int_3^7 e^{x^2} x\,dx$$

This is a definite integral so it is a fancy way to write some specific number.

Let $u = x^2$ so that $du = 2x\,dx$
and $u = 9$ when $x = 3$ and $u = 49$ when $x = 7$.

Hence:

$$\int_3^7 e^{x^2} x\,dx = \int_9^{49} e^u \frac{1}{2} du = \left[\frac{1}{2} e^u\right]_9^{49}$$

$$= \frac{1}{2}(e^{49} - e^9) = 9.537\ x\ 10^{20}$$

There are two other ways to do this question.

(1) Back substitute and use the original limits or

(2) Do it as an indefinite integral first then use this to evaluate the definite integral